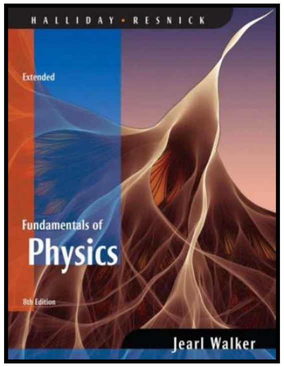
University Physics I

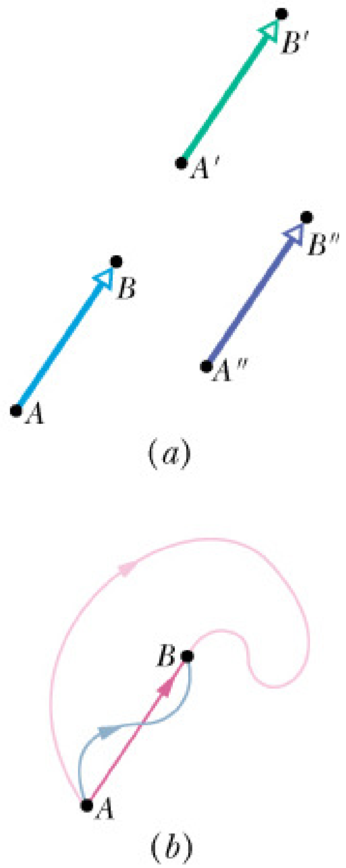


Week 2 : 3

Vectors

- In physics we have parameters that can be completely described by a number and are known as **scalars**. Temperature and mass are such parameters.
- Other physical parameters require additional information about direction and are known as **vectors**. Examples of vectors are displacement, velocity, and acceleration.
- Our goal is to learn the basic mathematical language to describe vectors. In particular we will learn the following:
  - Geometric vector addition and subtraction
  - Resolving a vector into its components
  - The notion of a unit vector
  - Addition and subtraction vectors by components
  - Multiplication of a vector by a scalar
  - The scalar (dot) product of two vectors
  - The vector (cross) product of two vectors

Vector Notation



An example of a vector is the displacement vector, which describes the change in position of an object as it moves from point *A* to point *B*. This is represented by an arrow that points from point *A* to point *B*. The length of the arrow is proportional to the displacement magnitude. The direction of the arrow indicated the displacement direction.

The three arrows from *A* to *B*, from *A′* to *B′*, and from *A″* to *B″*, have the same magnitude and direction. A vector can be shifted without changing its value if its length and direction are not changed.

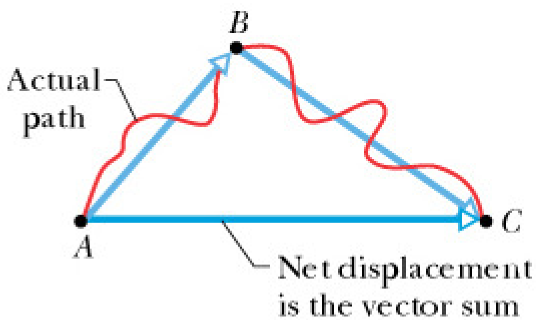
In books vectors are written in two ways:

Method 1: $\vec{a}$ (using an arrow above)

Method 2: **a** (using boldface print)

The magnitude of the vector is indicated by italic print: *a*.

Geometric Vector Addition



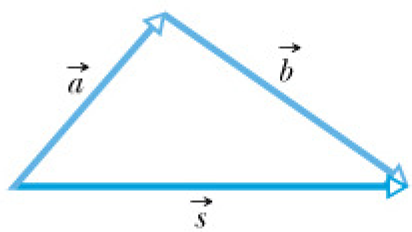
$\vec{s} = \vec{a} + \vec{b}$

Sketch vector $\vec{a}$ using an appropriate scale.

Sketch vector $\vec{b}$ using the same scale.

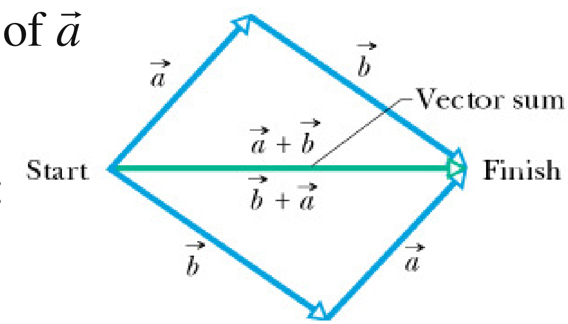
Place the tail of $\vec{b}$ at the tip of $\vec{a}$.

The vector $\vec{s}$ starts from the tail of $\vec{a}$ and terminates at the tip of $\vec{b}$.

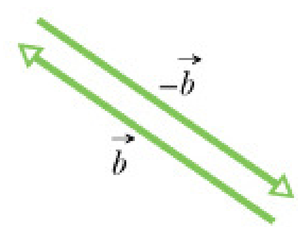


Vector addition is commutative:

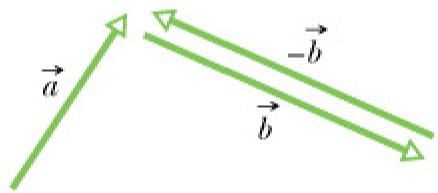
$\vec{a} + \vec{b} = \vec{b} + \vec{a}$



Negative $-\vec{b}$ of a given vector $\vec{b}$ has the same magnitude as $\vec{b}$ but opposite direction.

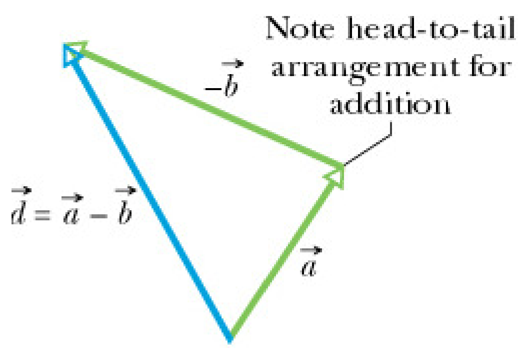


Geometric Vector Subtraction



$$\vec{d} = \vec{a} - \vec{b}$$

We write: $\vec{d} = \vec{a} - \vec{b} = \vec{a} + \left(-\vec{b}\right).$



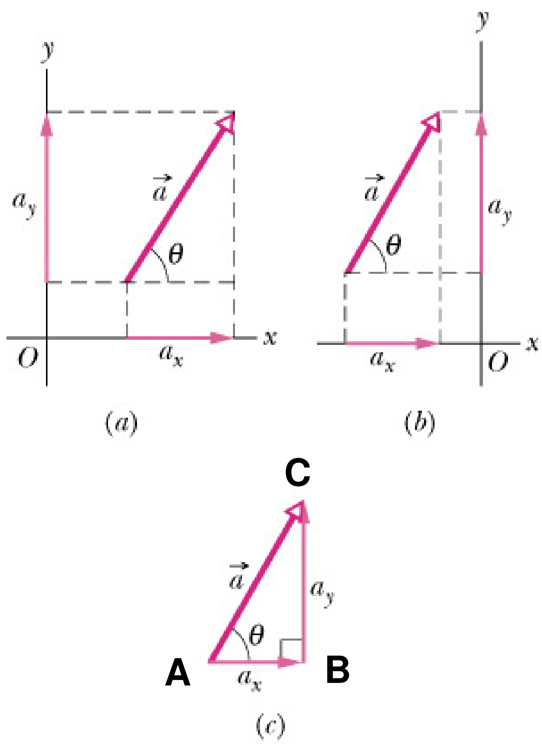
From vector $\vec{b}$ we find $-\vec{b}$.

Then we add $\left(-\vec{b}\right)$ to vector $\vec{a}$.

We thus reduce vector subtraction to vector addition, which we know how to do.

Note: We can add and subtract vectors using the method of components. For many applications this is a more convenient method.

Vector Components



A component of a vector along an axis is the projection of the vector on this axis. For example a_x is the projection of $\vec{a}$ along the x-axis. The component a_x is defined by drawing straight lines from the tail and tip of the vector $\vec{a}$ that are perpendicular to the x-axis.

From triangle ABC the x- and y-components of vector $\vec{a}$ are given by the equations

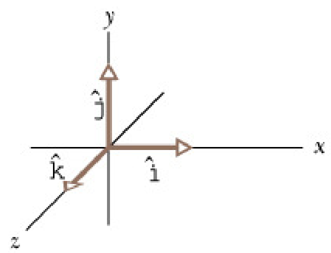
$a_x = a\cos\theta$ and $a_y = a\sin\theta$.

If we know a_x and a_y we can determine a and θ.

From triangle ABC we have:

$$a = \sqrt{a_x^2 + a_y^2} \quad \text{and} \quad \tan\theta = \frac{a_y}{a_x}.$$

Unit Vectors

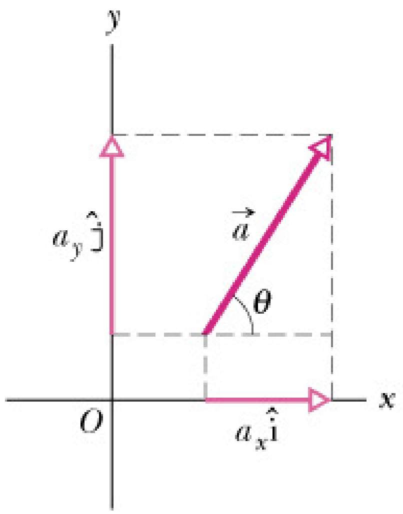


Unit vectors are used to express other vectors

For example vector $\vec{a}$ can be written as

$\vec{a} = a_x\hat{i} + a_y\hat{j}$.

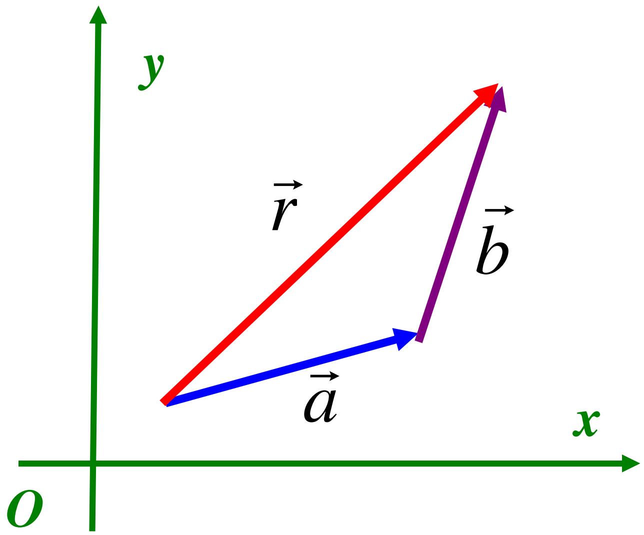
The quantities $a_x\hat{i}$ and $a_y\hat{j}$ are called the vector components of vector $\vec{a}$.



A unit vector is defined as a vector that has magnitude equal to 1 and points in a particular direction. Unit vectors lack units and their sole purpose is to point in a particular direction. The unit vectors along the x, y, and z axes are labeled $\hat{i}$, $\hat{j}$, and $\hat{k}$, respectively.

Vector Components

Adding Vectors



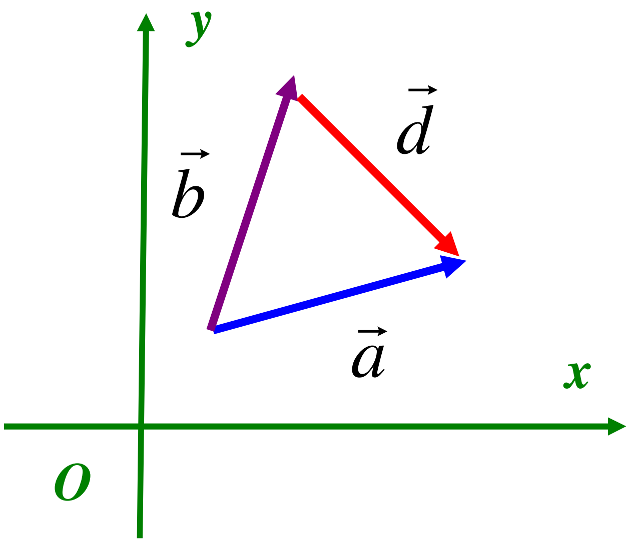
We are given two vectors $\vec{a} = a_x\hat{\text{i}} + a_y\hat{\text{j}}$ and $\vec{b} = b_x\hat{\text{i}} + b_y\hat{\text{j}}$.

We want to calculate the vector sum $\vec{r} = r_x\hat{\text{i}} + r_y\hat{\text{j}}$.

The components r_x and r_y are given by the equations

$r_x = a_x + b_x$ and $r_y = a_y + b_y$.

Subtracting Vectors



We are given two vectors $\vec{a} = a_x\hat{\text{i}} + a_y\hat{\text{j}}$ and $\vec{b} = b_x\hat{\text{i}} + b_y\hat{\text{j}}$.

We want to calculate the vector difference

$\vec{d} = \vec{a} - \vec{b} = d_x\hat{\text{i}} + d_y\hat{\text{j}}$.

The components d_x and d_y of $\vec{d}$ are given by the equations

$d_x = a_x - b_x$ and $d_y = a_y - b_y$.

Vector Multiplication

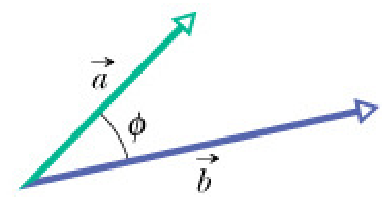
Multiplying a Vector by a Scalar

Multiplication of a vector $\vec{a}$ by a scalar s results in a new vector $\vec{b} = s\vec{a}$.

The magnitude b of the new vector is given by $b = |s|a$.

If $s > 0$, vector $\vec{b}$ has the same direction as vector $\vec{a}$.

If $s < 0$, vector $\vec{b}$ has a direction opposite to that of vector $\vec{a}$.



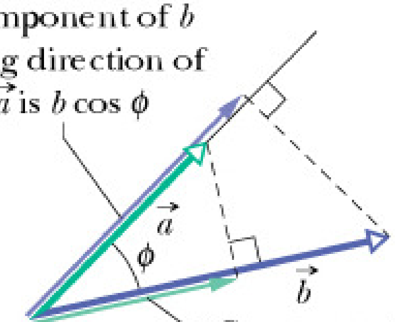
(a)

The Scalar Product of Two Vectors

The scalar product $\vec{a} \cdot \vec{b}$ of two vectors $\vec{a}$ and $\vec{b}$ is given by $\vec{a} \cdot \vec{b} = ab\cos\phi$. The scalar product of two vectors is also known as the "dot" product. The scalar product in terms of vector components is given by the equation

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z.$$

***Note: The Scalar Product is more often called the "DOT" Product.**



(b)

The Vector "Cross" Product

The Vector Product $\vec{c} = \vec{a} \times \vec{b}$ in Terms of Vector Components

$$\vec{a} = a_x \hat{\mathrm{i}} + a_y \hat{\mathrm{j}} + a_z \hat{\mathrm{k}},\ \ \vec{b} = b_x \hat{\mathrm{i}} + b_y \hat{\mathrm{j}} + b_z \hat{\mathrm{k}},\ \ \vec{c} = c_x \hat{\mathrm{i}} + c_y \hat{\mathrm{j}} + c_z \hat{\mathrm{k}}$$

The vector components of vector $\vec{c}$ are given by the equations

$$c_x = a_y b_z - a_z b_y,\quad c_y = a_z b_x - a_x b_z,\quad c_z = a_x b_y - a_y b_x.$$

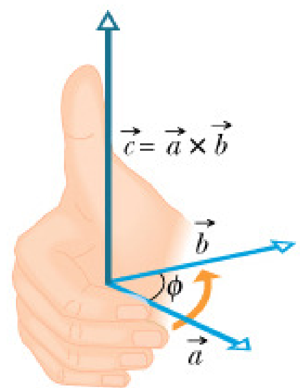
Note: Those familiar with the use of determinants can use the expression

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

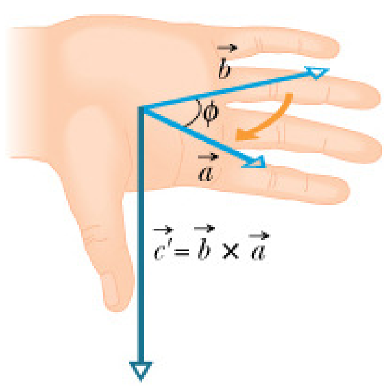
Note: The order of the two vectors in the cross product is important:

$$\vec{b} \times \vec{a} = -\left(\vec{a} \times \vec{b}\right).$$

The Right Hand Rule



(a)



The vector product $\vec{c} = \vec{a} \times \vec{b}$ of the vectors $\vec{a}$ and $\vec{b}$ is a vector $\vec{c}$.

The magnitude of $\vec{c}$ is given by the equation $c = ab \sin\phi$.

The direction of $\vec{c}$ is perpendicular to the plane P defined by the vectors $\vec{a}$ and $\vec{b}$.

The sense of the vector $\vec{c}$ is given by the right-hand rule:

a. Place the vectors $\vec{a}$ and $\vec{b}$ tail to tail.

b. Rotate $\vec{a}$ in the plane P along the shortest angle so that it coincides with $\vec{b}$.

c. Rotate the fingers of the right hand in the same direction.

d. The thumb of the right hand gives the sense of $\vec{c}$.

The vector product of two vectors is also known as the "cross" product.

Activity – Measuring 2D Vectors

- In this activity
  - You will construct real position vectors to various points on your work table and then determine the displacement vectors between these points.
  - Each table should split as evenly as possible into two teams, “A”, and “B”.
  - Each team will measure the components of three position vectors and determine corresponding displacement vectors.

Name(s) and table number ____

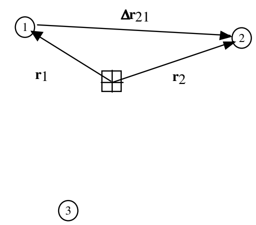
Measuring 2DVectors

In this activity you will construct real position vectors to various points on your work table and then determine the displacement vectors between these points. Each table will split as evenly as possible into two teams, “A”, and “B”. Each team will measure the components of three position vectors and determine corresponding displacement vectors.

Set up

On your table, are 3 colored dots labeled 1, 2, 3. These represent the points for which you will find position vectors. In addition there are two post-it notes with Coordinate System Origins marked A (for Team A) and B (for Team B.)

Procedure and measurements



Using a meter stick, your team will measure the *x*- and *y*-components of the position vectors for each of the three points using the coordinate system origin for your team. Measure as accurately as you can and estimate the uncertainties.

Next use your position vector measurements, $\vec{r}_1$, $\vec{r}_2$, $\vec{r}_3$ to determine the components of the three displacement vectors $\Delta\vec{r}_{21} = \vec{r}_2 - \vec{r}_1$, $\Delta\vec{r}_{32} = \vec{r}_3 - \vec{r}_2$, $\Delta\vec{r}_{13} = \vec{r}_1 - \vec{r}_3$. For each component of each displacement, calculate the measurement uncertainty, using the error propagation rules.

Note: Calculations called for on the following results page should be shown on separate pieces of paper and not squeezed onto the results page.

Team _________

Position	x (cm)			y (cm)		
$\vec{r}_1$		±			±	
$\vec{r}_2$		±			±	
$\vec{r}_3$		±			±	