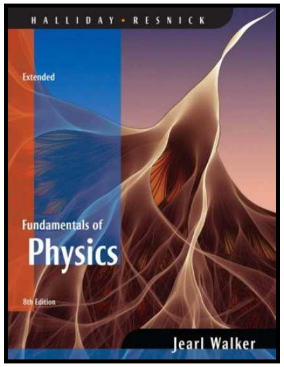
Workshop Physics 1017 - 311

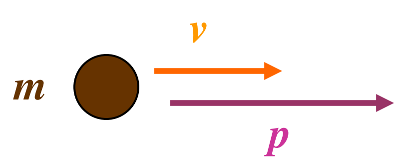
University Physics I



Week 9 : Day 2

RIT

Linear Momentum



Linear momentum $\vec{p}$ of a particle of mass m and velocity $\vec{v}$ is defined as $\vec{p} = m\vec{v}$.
The SI unit for linear momentum is the kg.m/s.

$$\boxed{\vec{p} = m\vec{v}}$$

Below we will prove the following statement: The time rate of change of the linear momentum of a particle is equal to the magnitude of net force acting on the particle and has the direction of the force.

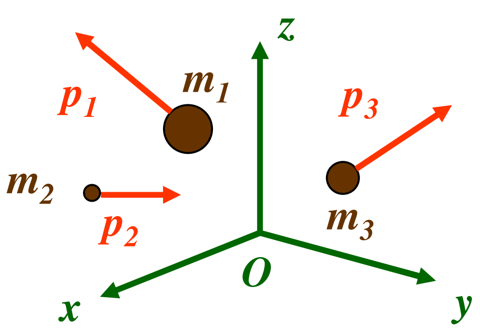
In equation form: $\vec{F}_{\text{net}} = \dfrac{d\vec{p}}{dt}$. We will prove this equation using Newton's second law:

$$\boxed{\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}}$$

$$\vec{p} = m\vec{v} \rightarrow \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a} = \vec{F}_{\text{net}}$$

This equation is stating that the linear momentum of a particle can be changed only by an external force. If the net external force is zero, the linear momentum cannot change:

Conservation of Linear Momentum



Consider a system of particles for which $F_{\text{net}} = 0$

$$\frac{d\vec{P}}{dt} = \vec{F}_{\text{net}} = 0 \rightarrow \vec{P} = \text{ Constant}$$

If no net external force acts on a system of particles, the total linear momentum $\vec{P}$ cannot change.

$$\begin{bmatrix} \text{total linear momentum} \\ \text{at some initial time } t_i \end{bmatrix} = \begin{bmatrix} \text{total linear momentum} \\ \text{at some later time } t_f \end{bmatrix}$$

The conservation of linear momentum is an important principle in physics. It also provides a powerful rule we can use to solve problems in mechanics such as collisions.

One Dimensional Linear, Elastic Collisions

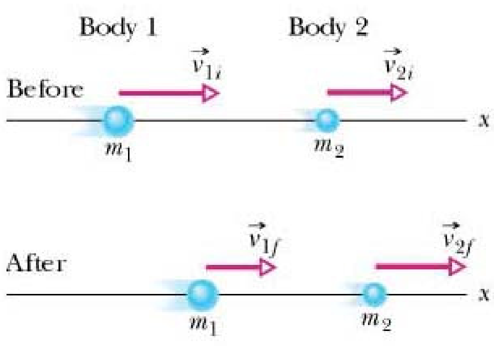
Both linear momentum and kinetic energy are conserved.

Linear momentum conservation: $m_1 v_{1i} + m_1 v_{2i} = m_1 v_{1f} + m_1 v_{2f}$ (eq. 1)

Kinetic energy conservation: $\frac{m_1 v_{1i}^2}{2} + \frac{m_1 v_{2i}^2}{2} = \frac{m_1 v_{1f}^2}{2} + \frac{m_2 v_{2f}^2}{2}$ (eq. 2)

We have two equations and two unknowns, v_{1f} and v_{1f}.

If we solve equations 1 and 2 for v_{1f} and v_{1f} we get the following solutions:



$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$$

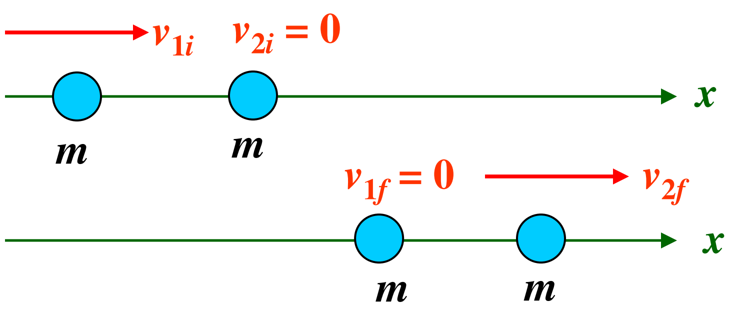
Special Case: Equal Masses

Below we examine several special cases for which we know the outcome of the collision from experience.

Equal masses $m_1 = m_2 = m$

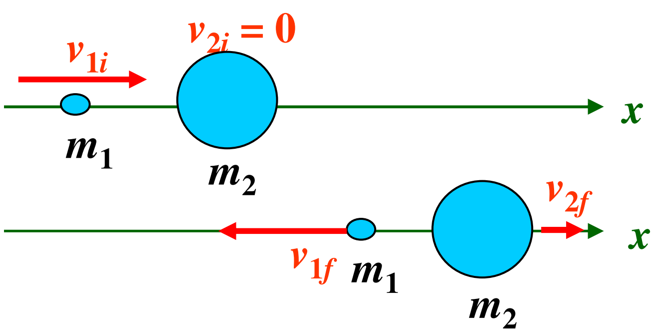
$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{m - m}{m + m} v_{1i} = 0$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2m}{m + m} v_{1i} = v_{1i}$$

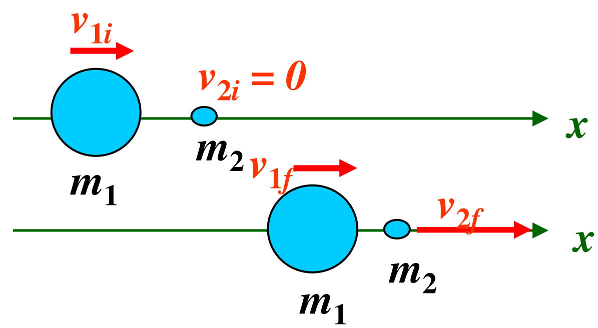


The two colliding objects have exchanged velocities.

Special Case: Massive Target or Projectile



A massive target



A massive projectile

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1} v_{1i} \approx -v_{1i}$$

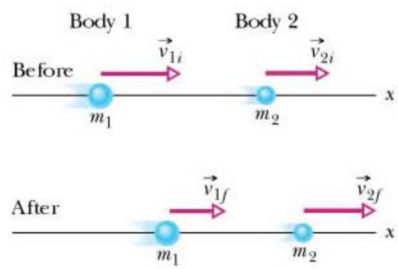
$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2\left(\frac{m_1}{m_2}\right)}{\frac{m_1}{m_2} + 1} v_{1i} \approx 2\left(\frac{m_1}{m_2}\right) v_{1i}$$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{1 - \frac{m_2}{m_1}}{1 + \frac{m_2}{m_1}} v_{1i} \approx v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2}{1 + \frac{m_2}{m_1}} v_{1i} \approx 2v_{1i}$$

Elastic and Inelastic Collisions

If the system is isolated, i.e., the net force $\vec{F}_{\text{net}} = 0$, linear momentum is conserved. The conservation of linear momentum is true regardless of the collision type. This is a powerful rule that allows us to determine the results of a collision without knowing the details. Collisions are divided into two broad classes: elastic and inelastic.



A collision is elastic if there is no loss of kinetic energy, i.e., $K_i = K_f$.

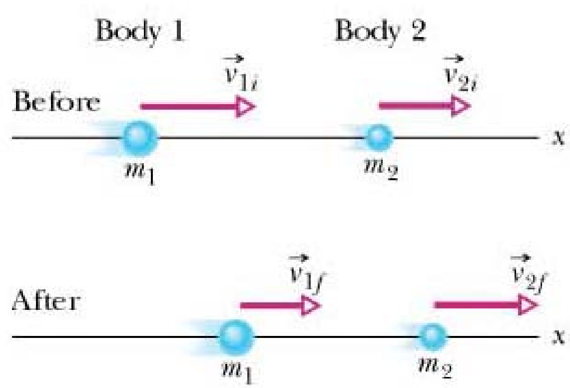
A collision is inelastic if kinetic energy is lost during the collision due to conversion into other forms of energy. In this case we have $K_f < K_i$.

A special case of inelastic collisions are known as completely inelastic. In these collisions the two colliding objects stick together and they move as a single body. In these collisions the loss of kinetic energy is maximum.

Inelastic Collisions

In these collisions the linear momentum of the colliding objects is conserved $\rightarrow$ $\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$.

$$m_1\vec{v}_{1i} + m_1\vec{v}_{2i} = m_1\vec{v}_{1f} + m_1\vec{v}_{2f}$$



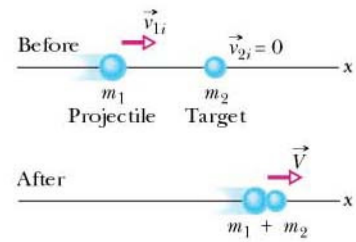
Momentum is Conserved but Energy is not Conserved in an Inelastic Collision

$$\cancel{\frac{m_1 v_{1i}^2}{2} + \frac{m_1 v_{2i}^2}{2} = \frac{m_1 v_{1f}^2}{2} + \frac{m_2 v_{2f}^2}{2}}$$

Completely Inelastic Collisions

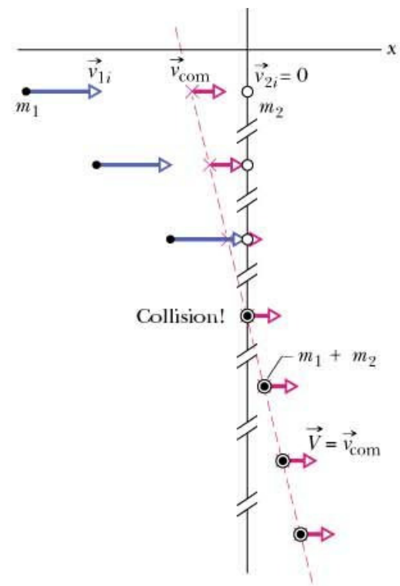
In these collisions the two colliding objects stick together and move as a single body. In the figure to the left we show a special case in which $\vec{v}_{2i} = 0. \rightarrow m_1 v_{1i} = m_1 V + m_1 V \rightarrow$

$$V = \frac{m_1}{m_1 + m_2} v_{1i}$$



The velocity of the center of mass in this collision is $\vec{v}_{\text{com}} = \dfrac{\vec{P}}{m_1 + m_2} = \dfrac{\vec{p}_{1i} + \vec{p}_{2i}}{m_1 + m_2} = \dfrac{m_1 \vec{v}_{1i}}{m_1 + m_2}$.

In the picture to the left we show some freeze-frames of a totally inelastic collision.



Linear Momentum for a System of Particles

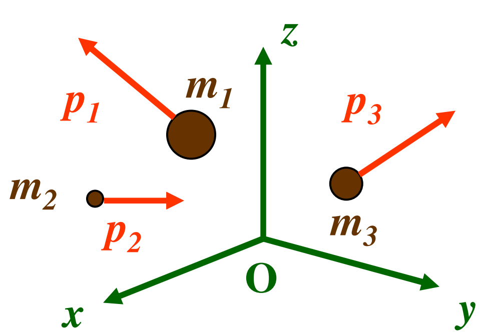
We define the linear momentum of a system of n particles as follows:

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots + \vec{p}_n = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots + m_n\vec{v}_n = M\vec{v}_{\text{com}}.$$

The linear momentum of a system of particles is equal to the product of the total mass M of the system and the velocity $\vec{v}_{\text{com}}$ of the center of mass.

The time rate of change of $\vec{P}$ is $\dfrac{d\vec{P}}{dt} = \dfrac{d}{dt}\left(M\vec{v}_{\text{com}}\right) = M\vec{a}_{\text{com}} = \vec{F}_{\text{net}}.$

The linear momentum $\vec{P}$ of a system of particles can be changed only by a net external force $\vec{F}_{\text{net}}$. If the net external force $\vec{F}_{\text{net}}$ is zero, $\vec{P}$ cannot change.



$$\boxed{\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots + \vec{p}_n = M\vec{v}_{\text{com}}}$$

Activity – Car and Truck Collisions

- Analyze 1-D Collisions
  - Elastic
  - Inelastic

Car-truck collisions

Momentum turns out to be enormously useful in solving problems that involve collisions, even when you don't know the details. Why? Because under very common circumstances, the total momentum of a set of colliding objects will be the same before and after they collide. In other words, **momentum is conserved**.

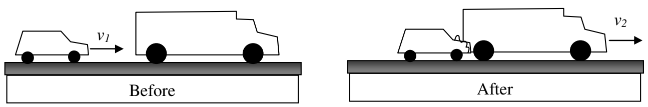
So, considering just 1-D motion for now, we can say

$$m_{A,1}v_{A,1} + m_{B,1}v_{B,1} + \ldots + m_{X,1}v_{X,1} = m_{A,2}v_{A,2} + m_{B,2}v_{B,2} + \ldots + m_{X,2}v_{X,2}$$

where subscripts A, B, C etc label different objects and 1 and 2 refer to times immediately before and immediately after the collision respectively.

Now consider the following cases:

1. Car crashes into stationary truck, they stick together



The car has mass m = 2000 kg and velocity v_1 = 20 m s^{-1}. The truck is initially stationary and has mass M =8000 kg.

What is the velocity, v_2, of the tangled wreckage immediately after the collision?

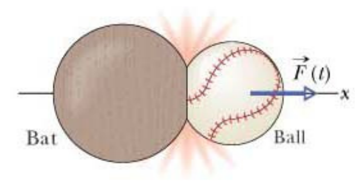
Is this an elastic, or inelastic collision (is kinetic energy conserved)?

RIT

Impulse and Momentum

Consider the collision of a baseball with a baseball bat.

The collision starts at time t_i when the ball touches the bat and ends at t_f when the two objects separate.



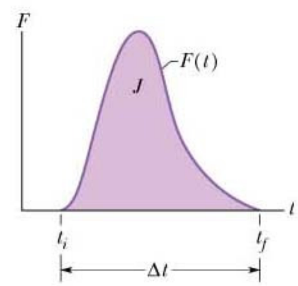
The ball is acted upon by a force $\vec{F}(t)$ during the collision.

The magnitude $F(t)$ of the force is plotted versus t in fig. a.

The force is nonzero only for the time interval $t_i < t < t_f$.

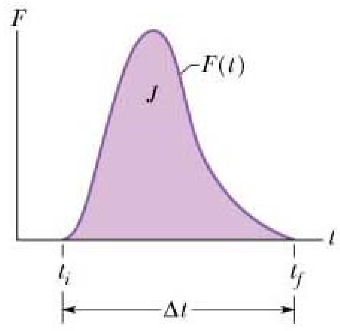
$\vec{F}(t) = \dfrac{d\vec{p}}{dt}$. Here $\vec{p}$ is the linear momentum of the ball,

$$d\vec{p} = \vec{F}(t)dt \rightarrow \int_{t_i}^{t_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)dt$$

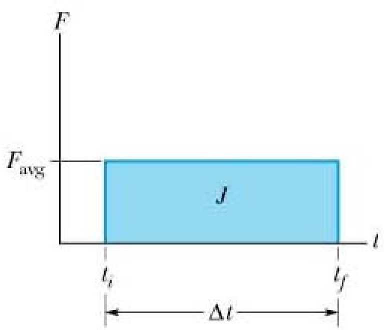


Impulse Calculations

$$\Delta p = J$$



(a)



(b)

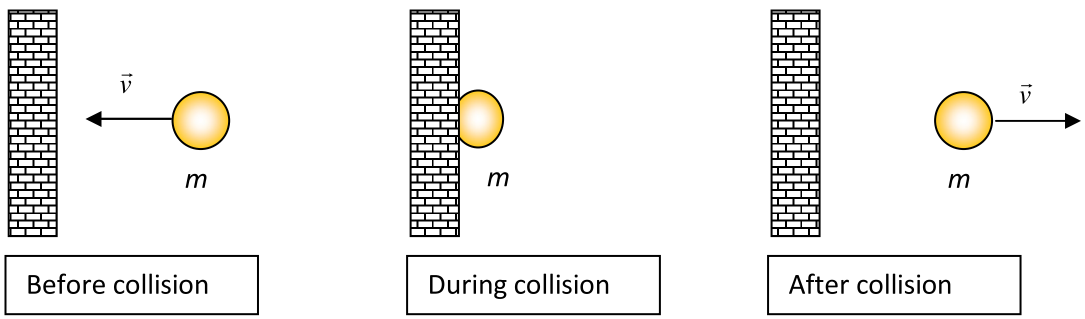
$\int_{t_i}^{t_f} \vec{F}(t)dt$ is known as the impulse $\vec{J}$ of the collision.

$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)dt$ The magnitude of $\vec{J}$ is equal to the area under the F versus t plot of fig. a $\rightarrow \Delta\vec{p} = \vec{J}$.

In many situations we do not know how the force changes with time but we know the average magnitude F_{ave} of the collision force. The magnitude of the impulse is given by $J = F_{\text{ave}}\Delta t$, where $\Delta t = t_f - t_i$.

Geometrically this means that the area under the F versus t plot (fig. a) is equal to the area under the F_{ave} versus t plot (fig. b).

Impulse on a Tennis Ball



a) What is the ball's initial momentum?
b) What ball's final momentum?
c) What is the impulse?
d) The ball is in contact with the wall for t=0.03 s. What is the average force on the ball during the collision?